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B.Sc Part - I

PAPER — II

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Trace the curve (Folium of Descartes)

$$x^3 + y^3 = 3axy.$$

Tracing: The equation of the curve is

$$x^3 + y^3 = 3axy \quad \text{— (1)}$$

The following points will suggest the tracing of the curve.

(i) If we interchange x and y in the equation (1) of the curve, we find that the equation (1) remain unaltered. Hence the curve is symmetrical about the line $y = x$.

(ii) We find that the equation (1) of the curve does not contain any constant term. Hence the curve passes through the origin.

(iii) To get the tangents to the curve at the origin, equate the lowest degree term, that is $3axy$ is zero.

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But $3a \neq 0$ $\therefore xy = 0$

$x = 0$ & $y = 0$

which are tangents to the curve at the Origin.

Hence there is a node (that is loop) at the Origin. That is two branches of the curve cross each other.

(IV) To get the asymptotes, rewrite the equation (1) as follows

$$(x+y)(x^2 - xy + y^2) = 3axy$$

The asymptote parallel to $x+y = 0$ is

$$x+y = \lim \frac{3axy}{x^2 - xy + y^2}$$

When $x \to \infty$ $y \to \infty$ and $\frac{y}{x} \to -1$;

$$\text{Or, } \quad x+y = \lim \frac{3a\left(\frac{y}{x}\right)}{1 - \frac{y}{x} + \left(\frac{y}{x}\right)^2}$$

$$= \frac{3a(-1)}{1-(-1)+(-1)^2} = -a$$

or, $x+y+a=0$

Also x^2-xy+y^2 Can't be resolved into rational factors, therefore there are no more real asymptotes.

(v.) When $x=0$, $y=0$

and when $y=0$, $x=0$

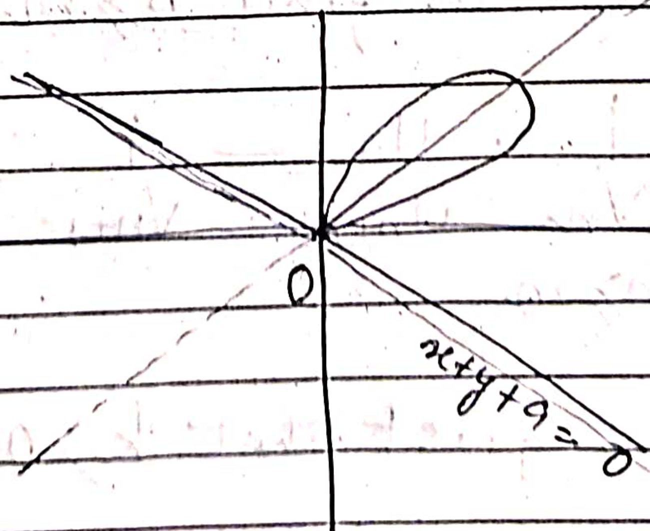
Hence the curve meets the axes only at the Origin

Again, when $y=x$

$$2x^3 = 3ax^2$$

$\therefore x=0$ or $x=\frac{3a}{2}$

that is, the curve meets the line $y=x$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$



These points suggest that the Shape of the curve is as shown in the given figure above.